DJ-003-001618 Seat No. ________

# B. Sc. (Sem. VI) (CBCS) Examination

**March-2022**

## Optimization and Numerical Analysis-II : BSMT-602(A)

*(Old Course)*

**Faculty Code : 003**
**Subject Code : 001618**

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

**Instructions :** (1) All the questions are compulsory.
(2) Right hand side digit indicates the mark.

1 Answer the following questions in short : **20**

(1) Write the full form of MODI method.

(2) The dual of dual is a prime problem. [True/False]

(3) The unbalanced transportation problem can be balanced by adding a dummy supply row or a demand column as per the need. [True/False]

(4) LCM is to find initial solution. [True/False]

(5) What is the special case of Bessel's formula ?

(6) If $f(x)=x^3$, then find $f(1,3,5,7)$

(7) Find the value of $\int_3^6 \frac{1}{x}dx$ by trapezoidal rule.

(8) Which formula is mean of Gauss's forward and Gauss's backward interpolation formula ?

(9) In Simpson's $\frac{1}{3}$ rule what is the form of the function $f(x)$?

(10) Divided difference is applicable for interpolation where the arguments are unequal. [True/False]

(11) An optimum solution does not necessarily use up all the limited resources available [True/False]

(12) Which type of differential equation can be solved using the Picard's method ?

(13) State the fundamental theorem of LPP.

(14) Define : *Slake variables*.

(15) Define : *Basic feasible solution*

(16) Define : *Concave funcation*.

(17) Define : *Unbounded Solution*

(18) What is inerpolation ?

(19) State Gauss-Backward interpolation Formula.

(20) State relation between divided difference and forward difference.

**2** (a) Attempt any **three** : **6**

(1) Maximize $Z = 11x_1 + 9x_2$, Subject to $3x_1 + 2x_2 \leq 8, 2x_1 + 3x_2 \leq 7$, where $x_1, x_2 \geq 0$ using graphical method.

(2) Write the full forms of VAM and NWCM.

(3) Explain the prime-dual relationship.

(4) State the general mathematical form of LPP.

(5) Define : *Extreme point and Optimal Solution.*

(6) Give the tabular form of transportation problem.

(B) Attempt any **three** : **9**

(1) Determine an initial solution of the given transportation problem using LCM.

*Destination*

| | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | *Supply* |
|---|---|---|---|---|---|---|
| *Sources* | $S_1$ | 21 | 16 | 15 | 3 | 11 |
| | $S_2$ | 17 | 18 | 14 | 23 | 13 |
| | $S_3$ | 32 | 27 | 18 | 41 | 19 |
| | *Demand* | 6 | 6 | 8 | 23 | |

(2) Find dual of the given LPP.

$Min\ Z = 7x_1 + 3x_2 + 8x_3$

Subject to

$8x_1 + 2x_2 + x_3 \geq 3$

$3x_1 + 6x_2 + 4x_3 \geq 4$

$4x_1 + x_2 + 5x_3 \geq 1$

$x_1 + 5x_2 + 2x_3 \geq 7$

*and* $x_1, x_2, x_3 \geq 0$

(3) Explain the steps of VAM to find intial solution of transportation problem.

(4) Explain mathematical formulation of an assignment problem.

(5) Explain the steps of two-phase method to solve the LPP.

(6) Explain the steps of Big-M method.

(C) Attempt any **Two** : **10**

(1) Use penalty method to solve the following LP problem.

$Min\ Z = 5x_1 + 3x_2$

*Subject to*

$2x_1 + 4x_2 \leq 12$

$2x_1 + 2x_2 = 10$

$5x_1 + 2x_2 \geq 10$

*and*

$x_1, x_2, x_3 \geq 0$

(2) Solve the following LPP by using the two-phase simplex method.

$Max\ Z = 2x_1 + 3x_2 + x_3$

*Subject to*

$x_1 + x_2 + x_3 \le 40$

$2x_1 + x_2 - x_3 \le 10$

$-x_2 + x_3 \ge 10$

*and*

$x_1, x_2, x_3 \ge 0$

(3) Solve the following transportation problem using MODI method.

TO

| | | $W_1$ | $W_2$ | $W_3$ | *Supply* |
|---|---|---|---|---|---|
| *From* | $F_1$ | 2 | 7 | 4 | 5 |
| | $F_2$ | 3 | 3 | 1 | 8 |
| | $F_3$ | 5 | 4 | 7 | 7 |
| | $F_4$ | 1 | 6 | 2 | 14 |
| | *Demand* | 7 | 9 | 18 | |

(4) Solve the following assignment problem.

Subordinates

| | | I | II | III | IV |
|---|---|---|---|---|---|
| | A | 8 | 26 | 17 | 11 |
| Tasks | B | 13 | 28 | 4 | 26 |
| | C | 38 | 19 | 18 | 15 |
| | D | 19 | 26 | 24 | 10 |

(5) Explain steps of Hungarian Method to solve the assignment problem.

3 (A) Attempt any **Three** : **6**

(1) If $f(x) = \frac{1}{x^2}$, then find $f(a, b, c, d)$.

(2) Evaluate $\int_0^{10} \frac{1}{1+x^2} dx$ by trapezoidal method.

(3) Give the central differences table for the arguments (1, 1), (2, 4), (3, 6), (4, 10), (5, 17).

(4) In usual notation prove that

$$D = \frac{1}{h}\left[\Delta \rightarrow \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \frac{1}{4}\Delta^4 + ...\right]$$

(5) Prove that the value of any divided difference is independent of the order of the arguments.

(6) Write Lagrange's interpolation formula.

(B) Attempt any **Three** : **9**

(1) Evaluate $\int_1^3 x^2$ using Simpson's $3/8$ rule.

(2) Find $y_{35}$ for the data,

| $x$ | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|
| $y$ | 600 | 512 | 439 | 346 | 243 |

(3) Find $y(0.2), y(0.4)$ *and* $y(0.6)$ by Euler's method

$if \frac{dy}{dx} = 2x + y, y(0) = 1$

(4) Derive Newton's forward interpolation formulae form divided difference imterpolation formula.

(5) Derive Simpson's $1/3$ Rule.

(6) Solve $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$ by Taylor's series method.

(C) Attempt any **Two** : **10**

(1) If $\frac{dy}{dx} = 2e^x - y, y(0) = 2, y(0.1) = 2.010,$

$y(0.2) = 2.040, y(0.3) = 2.090$, then find y(0.4) correct up to three decimal places applying milne's predictor method.

(2) Obtain the value of $f'(90)$ using Striling's formula for the following data.

| $x$ | 60 | 75 | 90 | 105 | 120 |
|---|---|---|---|---|---|
| $f(x)$ | 28.2 | 38.2 | 43.2 | 40.9 | 37.7 |

(3) Using Gauss's forward interpolation formula find the value of $f(337.5)$ from the following data.

| $x$ | 310 | 320 | 330 | 340 | 350 | 360 |
|---|---|---|---|---|---|---|
| $f(x)$ | 2.4914 | 2.5051 | 2.5185 | 2.5315 | 2.5441 | 2.5563 |

(4) Derive Gauss backward interpolation formula and hance deduce Simpson's 3/8 rule.

(5) Derive general quadrature formula.